


POTENTIALS OF ENVELOPING CROSSED HELICAL GEARS

A 3D CAD rendering of two crossed helical gears. The gears are shown in a perspective view, with one gear in the foreground and another behind it. The teeth are clearly defined, and the meshing area is visible. The background is a light blue gradient with faint diagonal lines.

Results of this study support the potential application of enveloping crossed axis helical gears in moderate to highly loaded gear drives.

By DR. ALEXANDER L. KAPELEVICH, YEFIM I. SAFRIS, DR.-ING. JOACHIM THOMAS, DR. CLAUDE GOSSELIN, and DR.-ING. WOLF WAGNER

Crossed axis helical gears (Figure 1), otherwise called screw gears, have been known as long as parallel axis helical gears for at least two centuries. However, unlike parallel axis helical gears, their application is limited by insufficient load-carrying capacity due to low wear resistance (Figure 2) resulting from tooth sliding and point tooth contact which leads to high Hertzian contact stress.

1 INTRODUCTION

This study focuses on minimizing Hertzian contact stress and to identify a manufacturing-friendly gear tooth geometry that maximizes the load-carrying capacity of crossed axis helical gears.

According to [1], a ZI involute helicoid worm and a helical gear make point contact, while a ZI involute helicoid worm and a conjugated enveloping worm wheel are in line contact. The proposed enveloping helical gear resembles a worm wheel engaged with a helical gear instead of a ZI involute helicoid worm (Figure 3).

The worm wheel teeth are machined by a generating worm (worm gear hob), similar to the mating ZI worm. The enveloping helical gear tooth geometry can be defined the same way. However, the worm wheel is typically made of copper alloy (brass or bronze) that conforms during run-in, compensating for gear tolerances, probable assembly misalignment, gear tooth, and body deflection.

The hardened steel enveloping helical gear requires tooth contact localization to avoid edge contact. Different options for tooth profile and lead crowning are applicable. This study considers two such options. The first one is when the generating helical gear has a higher number of teeth than the mating helical pinion, thereby providing point contact, i.e., localized contact. The second considered option is when the enveloping crossed helical gear is generated by a helical gear with the same number of teeth as the mating pinion, resulting in line tooth contact. In this case, localized point tooth contact is achieved through the mating helical pinion lead crowning.

2 HERTZ CONTACT STRESS DEFINITION BY ROARK'S FORMULAS

Modern gear design standards use Hertzian pressure as the basis for contact stress calculations [2, 3]. The same methodology applies to defining the contact stress of enveloping crossed helical gears, using Roark's formulas for Hertzian stress and strain calculations [4].

For the conventional and localized contact enveloping crossed axis helical gears, the general case of two elastic bodies in point contact is applicable (Figure 4).

The $1/R_1$ and $1/R_1'$ are principal curvatures of body 1 and $1/R_2$ and $1/R_2'$ of body 2. The principal curvatures of each body are mutually perpendicular. The radii are positive if the center of curvature is within the given body, i.e., the surface is convex, and negative otherwise.

The major c and minor d contact ellipsis semiaxes, and S contact area are calculated in Equations 1:

$$c = \alpha \sqrt[3]{PK_D C_E}, \quad d = \beta \sqrt[3]{PK_D C_E}, \quad S = \pi cd. \quad \text{Equation 1}$$

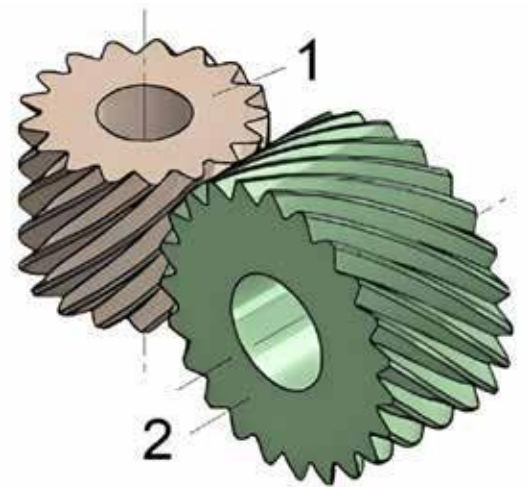


Figure 1: Crossed helical gear pair; 1 – helical pinion, 2 – helical gear.



Figure 2: The tooth flank wear pattern of a crossed axis helical gear under excessive load (Courtesy of Indal Technologies, Curtiss-Wright division).

The maximum Hertzian contact stress is calculated in Equation 2:

$$\sigma_{c\max} = \frac{1.5P}{c} = C_{SC} \sqrt[3]{P}, \quad \text{Equation 2}$$

The material property factor is calculated in Equation 3:

$$C_E = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}, \quad \text{Equation 3}$$

Where:

E_1 and E_2 , and ν_1 and ν_2 - moduli of elasticity and Poisson ratios of the body 1 and 2 materials.

The geometry factor is calculated in Equation 4:

$$K_D = \frac{1.5}{1/R_1 + 1/R_2 + 1/R_1' + 1/R_2'}, \quad \text{Equation 4}$$

The contact stress factor C_{SC} is calculated in Equation 5:

$$C_{SC} = \frac{1.5}{\alpha\beta\sqrt[3]{K_D^2 C_E^2}}. \quad \text{Equation 5}$$

The α , β , and γ coefficients are from Table 1 [4] in which Equation 6 is used:

$$\cos\theta = \frac{K_D}{1.5} \sqrt{\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)^2 + \left(\frac{1}{R_2} - \frac{1}{R_2'}\right)^2 + 2\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)\left(\frac{1}{R_2} - \frac{1}{R_2'}\right)\cos 2\phi}. \quad \text{Equation 6}$$

Table 1 is inconvenient for practical calculations. Instead, a spline interpolation function defines the α and β coefficients.

Figure 5 presents the helical pinion and gear tooth flanks in contact; the minor (Plane 1) and major (Plane 2) curvature radii cross-sections are shown at the pitch contact point of the conventional (a) and localized enveloping (b) crossed helical gear pairs.

Figures 6a and 6b show Plane 1 of Figures 5a and 5b with the minor curvature radii R_1 and R_2 cross-sections. Plane 1 is normal to the helical tooth spiral lines of the mating gears at the contact pitch point. The dashed lines represent the virtual spur gear tooth tip diameters, indicating that the conventional crossed axis helical gear cross-section (a) resembles an external spur gear mesh with convexo-convex tooth contact, while the enveloping crossed axis helical gear cross-section (b) looks like an internal spur gear mesh with the convexo-concave tooth contact.

The minor radii of curvature R_1 and R_2 at the pitch point lay at the same normal plane to the tooth lines of both mating gears. It makes the angle ϕ between the radius R_1 plane and the radius R_2 plane equal to $\phi = 0^\circ$. Such location of the minor radii results in maximum Hertzian contact stress.

For the conventional crossed axis helical gear pair, the pitch point minor radii of curvature of the driving helical pinion and driven helical gears are calculated in Equations 7:

$$R_1 = \frac{d_1 \sin \alpha_t}{2 \cos \beta_k} \quad \text{and} \quad R_2 = \frac{d_2 \sin \alpha_t}{2 \cos \beta_k}, \quad \text{Equation 7}$$

Where:

β_b - helix angle at the base diameters of both mating helical gears.

α_t - transverse pressure angle.

α - normal pressure angle.

β - helix angle at the pitch diameters of both mating helical gears.

For the localized enveloping crossed axis helical gear pair, the pitch point minor radius of curvature R_1 of the driving helical pinion

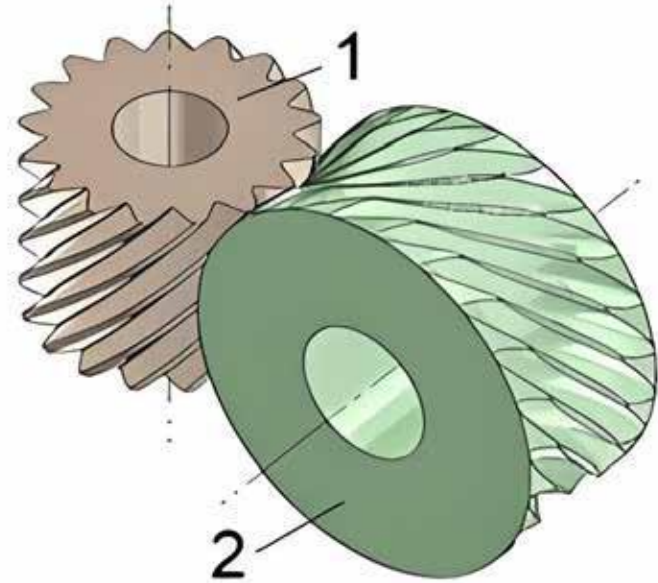


Figure 3: Enveloping crossed axis helical gear pair; 1 – helical pinion, 2 – enveloping helical gear

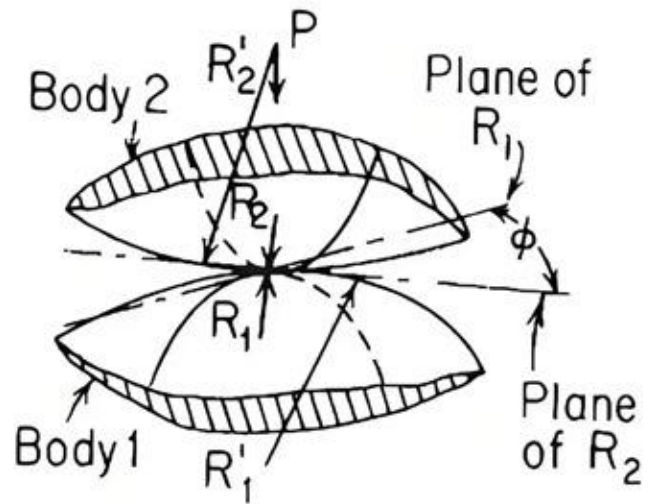


Figure 4: General case of two bodies in contact [4]; P – normal contact load, R_1 and R_1' – contact point minor and major principal radii of curvature for body 1, and R_2 and R_2' for body 2; ϕ – angle between plane of radius R_1 and plane of radius R_2 .

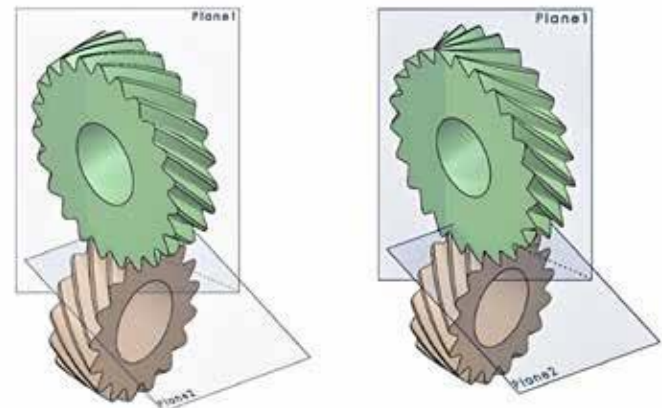


Figure 5: Minor (Plane 1) and major (Plane 2) curvature radii cross-sections at pitch point; a – conventional and b – enveloping crossed helical gears.

$\cos \theta$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.75
α	1.000	1.070	1.150	1.242	1.351	1.486	1.661	1.905	2.072
β	1.000	0.936	0.878	0.822	0.769	0.717	0.664	0.608	0.578

$\cos \theta$	0.80	0.85	0.90	0.92	0.94	0.96	0.98	0.99
α	2.292	2.600	3.093	3.396	3.824	4.508	5.937	7.774
β	0.544	0.507	0.461	0.438	0.412	0.378	0.328	0.287

Table 1

Gear	Driving pinion	Driven gear
Gear type	Helical	Helical or enveloping helical
Number of teeth	17	23
Normal module, mm	1.5	1.5
Shaft angle	90°	
Normal pressure angle	25°	25°
Helix angle	45°	45°
Pitch diameter, mm	36.062	48.790
Outer diameter, mm	39.062	51.790*
Root diameter, mm	32.312	45.040*
Face width, mm	14.0	14.0
Center distance, mm	42.426	
Modulus of Elasticity, MPa	206000	206000
Poisson Ratio	0.3	0.3

*at the throat section for the enveloping helical gear

Table 2

Crossed axis helical gear pair	Conventional		Enveloping		Enveloping	
	Driving pinion	Driven gear	Driving pinion	Driven gear	Driving pinion	Driven gear
Gear	Helical	Helical	Helical	Enveloping Helical	Helical	Enveloping Helical
Tooth shape	Helical	Helical	Helical	Enveloping Helical	Helical	Enveloping Helical
Number of teeth	17	23	17	23	17	23
Number of teeth of generating gear	-	-	-	18	-	17
Tooth flank profile crowning, mm	-	-	-	0.004	-	-
Tooth flank lead crowning, mm	-	-	-	0.040	0.025	-
Minor curvature radius at pitch point, mm	12.931	17.495	12.931	-68.805*	12.93	-56.94*
Major curvature radius at pitch point, mm	84.558	112.001	84.558	-239.227*	77.77	-172.401*
Pitch point tooth contact	convexo-convex		convexo-concave		convexo-concave	
Initial pinion torque T_1 , Nm	5.00		5.00		5.00	
Initial normal pitch point load P , N	433		433		433	
Initial contact ellipsis area S_e , mm ²	0.413		0.728 (+76.3%)		0.757 (+83.3%)	
Hertz contact stress, MPa	1571		892 (-43.2%)		858 (-47.0%)	
Adjusted pinion torque T_{a1} , Nm	5.00		27.27		30.68	
Adjusted normal pitch point load P_a , N	433		2361.6		2656.4	
Adjusted contact ellipsis area S_e , mm ²	0.413		2.255 (+446%)		2.536 (+514%)	
Hertz stress under adjusted torque, MPa	1571		1571		1571	
Load-carrying capacity factor	1.0		5.46		6.14	

*concave curvatures have negative values

Table 3

is defined by Equation 7, while the minor radius of curvature R_2 at the pitch point is determined using the graphical-analytical method in Plane 1 (Figure 5b) of the enveloping helical 3D gear model.

Figures 7a and 7b show Plane 2 of Figures 5a and 5b with the major curvature radii R_1' and R_2' cross-sections. Plane 2 is simultaneously perpendicular to Plane 1 and the mating gears tooth profiles at the contact pitch point. The radii R_1' and R_2' are also defined using the graphical-analytical method in Plane 2 (Figures 5a and 5b) of the of

the conventional and enveloping helical 3D gear models.

Figures 6b and 7b show the concave minor and major curvatures of the enveloping helical gear, which result in the convexo-concave tooth contact with the helical pinion.

Table 2 presents the basic geometry parameters and material properties of the conventional and enveloping crossed axis

helical gears. Table 3 shows the results of the Hertzian contact stress analysis using Roark's Formulas at the pitch point of the conventional crossed axis helical gears and enveloping crossed axis helical gears with the localized point tooth contact, formed by the 18-tooth generating gear, and by the 17-tooth generating gear. The tooth flank of the enveloping helical gear, formed by the 18-tooth generating gear, results in 0.004 mm profile crowning and 0.040 mm lead crowning. The mating helical pinion tooth flank has no crowning. The tooth flank of the enveloping helical gear formed by the 17-tooth generating gear tooth flank has no crowning, but its mating helical pinion tooth flank has 0.025 mm lead crowning. The Hertz contact stress and contact ellipsis area of the conventional crossed axis helical gear pair is considered to be 100%.

Figure 8 illustrates the contact ellipses of the conventional and enveloping crossed helical gears under 5.0 Nm pinion torque.

The load-carrying capacity factor F_{LC} is a ratio of the adjusted pinion torque T_{a1} to the pinion torque T_1 . Under the adjusted torque T_{a1} , the maximum Hertz contact stress of the enveloping crossed helical gear pair equals the conventional crossed helical gear maximum Hertz contact stress under the pinion torque T_1 . See Equation 8:

$$F_{LC} = \frac{T_{a1}}{T_1} \quad \text{Equation 8}$$

For Roark's formulas' definition of the Hertz contact stress (Table 3), the conventional and enveloping crossed axis helical gear maximum Hertzian contact stress at the pitch point is proportional to the cubic root of the normal tooth load P (Equation 2). Then, the load-carrying capacity factor F_{LC} for the conventional and enveloping crossed axis helical gear pairs with the point contact is calculated in Equation 9:

$$F_{LC} = \frac{P_a}{P} = \left(\frac{\sigma_{cmaxC}}{\sigma_{cmaxE}} \right)^3 \quad \text{Equation 9}$$

Where:

σ_{cmaxC} – maximum Hertzian contact stress of the conventional crossed axis helical gears.

σ_{cmaxE} – maximum Hertzian contact stress

of the enveloping crossed axis helical gears.

The contact ellipses of conventional and enveloping crossed helical gears under adjusted pinion torque T_{a1} are shown in Figure 9.

3 LOADED TOOTH CONTACT ANALYSIS (LTCA) OF CROSSED AXIS HELICAL GEARS

An LTCA computer program analyzes how the transmitted torque is shared between meshing gear tooth pairs and, from the resulting tooth loads, calculates the resulting contact and bending stresses.

Additional information resulting from the LTCA often includes sliding friction, sub-surface shear stress, lubricant film thickness, scoring, and scuffing probabilities. The BECAL [5, 6] and HyGEARS [7] software programs were used for the crossed axis helical gears LTCA to verify the contact stress reduction and the load-carrying capacity increase shown by the above Hertz contact stress analysis (Table 3).

3.1 BECAL LTCA

Since the late 1980s, the Institute of Geometry at TU Dresden has been developing methods for generating approximation surfaces for spiral bevel gears in collaboration with the Institute of Machine Elements (IMM). These efforts later evolved into a comprehensive program package called BECAL (BEvel gear CALculation), designed for rapid tooth contact analysis of bevel gears under load.

Initially, the approximation surfaces consisted exclusively of Bézier surfaces, which provided highly accurate representations of point clouds for classic spiral bevel gear teeth. These point clouds are generated using a tooth flank generator integrated into BECAL, which operates based on machine setting data from bevel gear machine tools. Local tooth deformations under load are determined using stiffness influence numbers, calculated via the Boundary Element Method (BEM) to achieve a practical load distribution. These values have been calibrated through experimental gear measurements and Finite Element Method (FEM) calculations. Local equivalent cylinders are employed for precise local Hertzian pressure calculations, with curvatures originally determined in the normal section direction.

To accommodate complex modifications in flank geometry, capabilities are introduced to import external point clouds, which are also supported in the STEP file format. Additionally, enhancements in influence coefficient calculations for tooth deformations have been made. While using the local equivalent cylinders perpendicular to the contact line, it is now possible to analyze various bevel gears independently of machine setting data and a lot of other gear types, including conventional and enveloping crossed axis helical gears [6].

Table 4 presents the BECAL contact stress diagrams and maximum stresses of the conventional and enveloping crossed-axis helical driven gears for the same 5 Nm driving pinion torque.


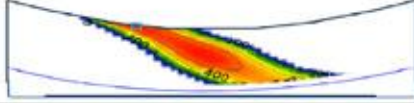
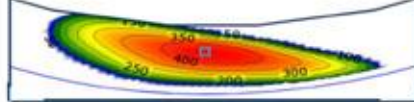
Driven helical gear	BECAL contact stress diagram	Maximum contact stress, MPa
Conventional		1340
Enveloping (formed by 18-tooth generating gear)		585
Enveloping (formed by 17-tooth generating gear)		433

Table 4

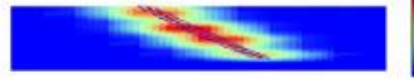
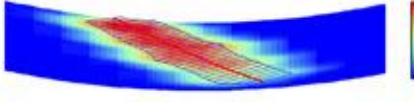
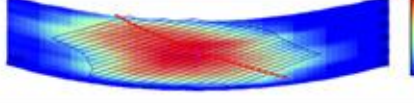
Driven helical gear	HyGEARS contact stress diagram	Maximum contact stress, MPa
Conventional		1466
Enveloping (formed by 18-tooth generating gear)		744
Enveloping (formed by 17-tooth generating gear)		570

Table 5

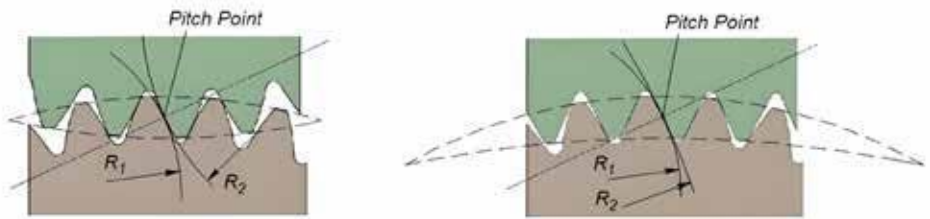


Figure 6: Minor curvature radii R_1 and R_2 cross-sections (Plane 1 of Figure 5); a – conventional and b – enveloping crossed helical gears; dash lines – virtual spur gear tooth tip diameters.

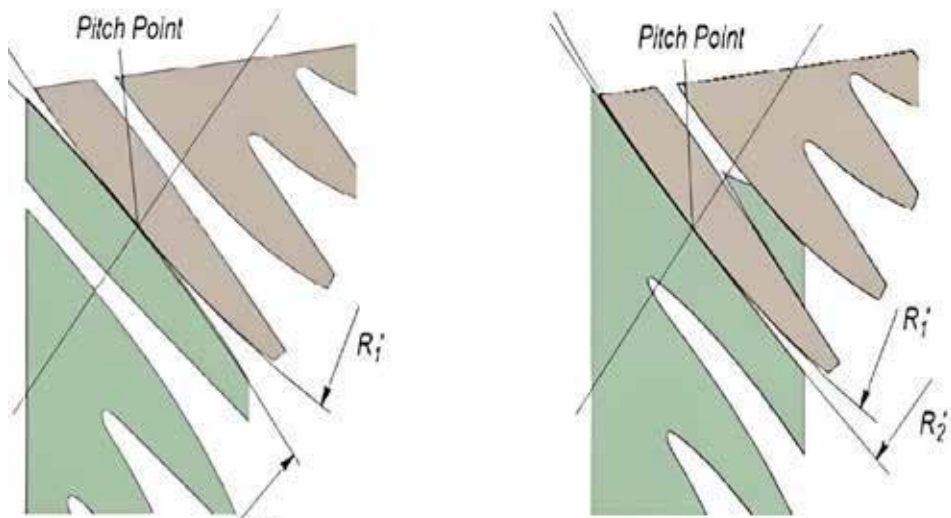


Figure 7: Major curvature radii R_1' and R_2' cross-sections (Plane 2 of Figure 5); a – conventional and b – localized enveloping crossed helical gears.

3.2 HYGEARS™ LTCA

HyGEARS™ is a self-contained gear design and analysis software in which the gear teeth

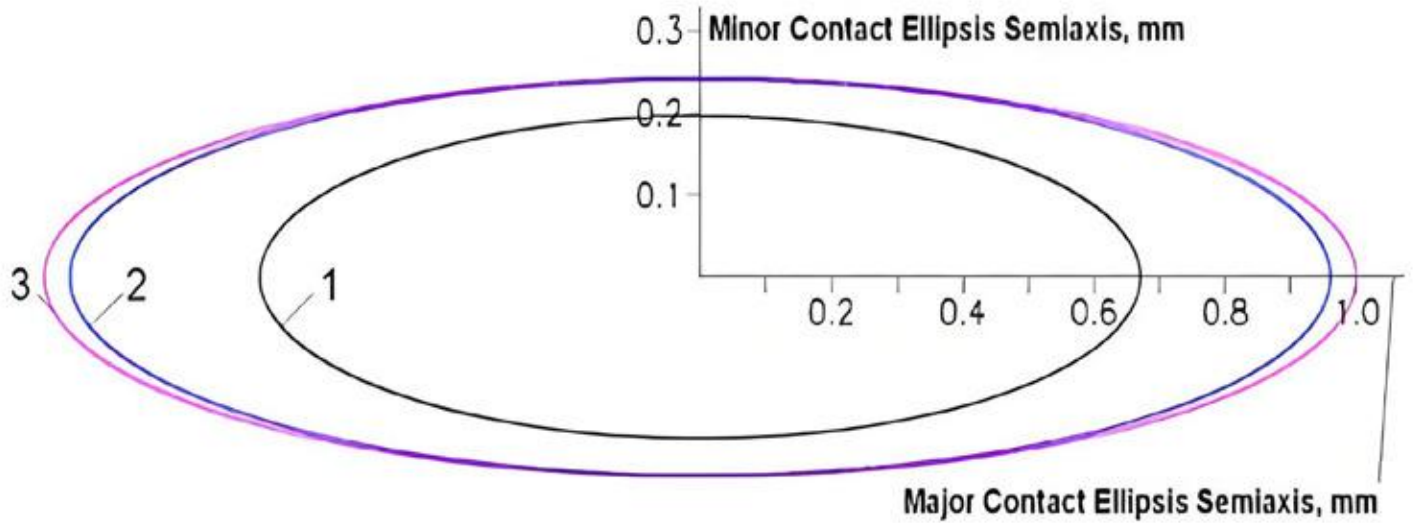


Figure 8: Contact ellipses under pinion torque $T_1 = 5.0$ Nm (Table 3); 1 – black, conventional crossed helical gears, 2 – blue, enveloping crossed helical gears (18-tooth generating gear), 3 – pink, enveloping crossed helical gears (17-tooth generating gear)

Crossed helical gear pair		Conventional	Enveloping	Enveloping
Pinion number of teeth		17	17	17
Gear number of teeth		23	23	23
Normal module, mm		1.5	1.5	1.5
Generating gear number of teeth		-	18	17
Maximum contact stress, MPa (at 5 Nm initial pinion torque)	Roark's Formulas	1571	892 (-43.2%)	858 (-47.0%)
	BECAL	1340	585 (-56.3%)	433 (-67.7%)
	HyGEARS	1466	744 (-49.2%)	570 (-61.1%)
Adjusted pinion torque*, Nm	Roark's Formulas	5.00	27.30	30.68
	BECAL	5.00	83.5	140.0
	HyGEARS	5.00	39.0	115.0
Load-carrying capacity factor	Roark's Formulas	1.0	5.46	6.14
	BECAL	1.0	16.7	28.0
	HyGEARS	1.0	7.8	23.0

*The adjusted pinion torque applied to the enveloping crossed axis helical gear pairs results in maximum contact stress equivalent to that of the conventional crossed axis helical gear pair's maximum contact stress, as defined for each calculation method.

Table 6

are digitized (or generated) from a set of cutter dimensions, machine settings, and machine movements such that one gets the exact same tooth flank topography that would be obtained if the same gear tooth was cut on a perfect machine with a perfect cutter [7].

For the enveloping helical gear, the shaping (generating) tool is either identical to the mating helical pinion, for line contact, or has a larger number of teeth for localized contact pattern on the tooth flank.

The following simplifications and assumptions are made:

- There is neither friction in the contact zone nor the presence of lubricant.
- Displacements due to tooth deflection, shearing, or contact deformation are sufficiently small that the tooth surface normal vector at the contact point is not affected.

The HyGEARS TCA computer program is used to obtain the kinematical characteristics of the analyzed gear sets.

The contact is Hertzian, and no boundary corrections are made for edge contact.

Tooth coupling effects are neglected.

In the HyGEARS LTCA, to establish the individual tooth loads at each meshing position, Equations 9 and 10 are solved using a Newton Raphson algorithm:

$$\frac{[\delta B_{i-1} + \Delta S_{i-1} + \Delta H_{i-1}]_t}{\|R_{ci-1}\|} = \frac{[\delta B_i + \Delta S_i + \Delta H_i]_t}{\|R_{ci}\|} = \frac{[\delta B_{i+1} + \Delta S_{i+1} + \Delta H_{i+1}]_t}{\|R_{ci+1}\|} \quad \text{Equation 10}$$

$$T_{app} = T_{i-1} + T_i + T_{i+1} \quad \text{Equation 11}$$

In other words:

The rotation (subscript t in Equation 11 implies the transverse plane) caused by the initial tooth to tooth separation Δ_S resulting from the TE, the contact deformation Δ_H and the bending deflection Δ_B must be the same between tooth pairs $i-1$, i and $i+1$.

The sum of the individual torque T_i applied to each meshing tooth pair must equal the externally applied torque T_{app} .

Table 5 presents the HyGEARS contact stress diagrams and maximum stresses of the conventional and enveloping crossed-axis helical driven gears for the same 5 Nm driving pinion torque.

4 ENVELOPING HELICAL GEAR CONTACT STRESS AND LOAD-CARRYING CAPACITY EVALUATION

Table 6 summarizes the maximum contact stress reduction and the load-carrying capacity increase for conventional and enveloping crossed axis helical gears, as determined by the Hertz equations, and the BECAL and HyGEARS software. The contact stress values for the conventional crossed axis helical gears, calculated using Hertz equations, the BECAL, and the HyGEARS LTCA, are assumed to be 100%.

The evaluation results show a noticeable trend in contact stress

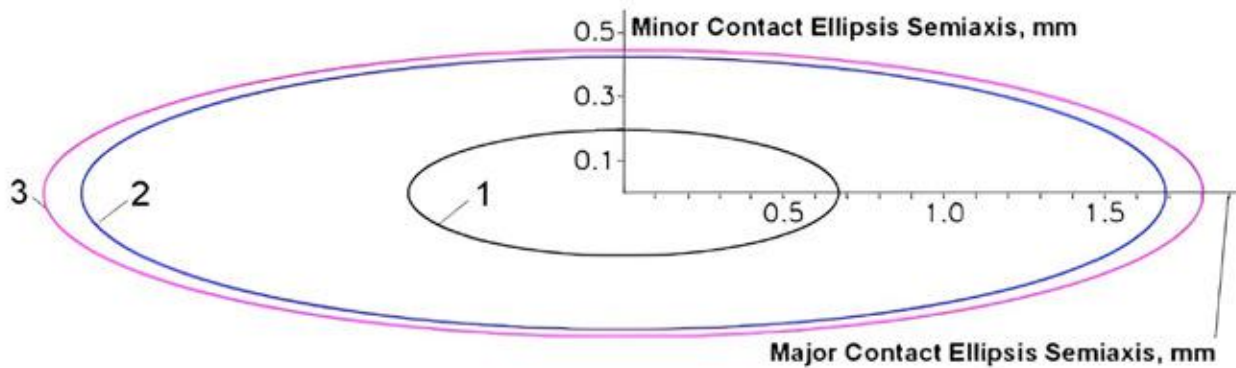


Figure 9: Contact ellipses under adjusted pinion torques T_{a1} (Table 3); 1 – black, conventional crossed helical gears, 2 – blue, enveloping crossed helical gears (18-tooth generating gear), 3 – pink, enveloping crossed helical gears (17-tooth generating gear).

reduction and increased load-carrying capacity for enveloping crossed axis helical gears compared to conventional crossed axis helical gears, despite considerable variance in maximum contact stress values depending on the calculation method.

5 ENVELOPING HELICAL GEAR MANUFACTURING METHODS

The presented enveloping helical gears evolved from the worm wheels, which are machined by the gear hob similar to the mating ZI worm. The generating gear used to define the enveloping helical gear tooth topography has a tooth count equal to or slightly greater than the mating helical pinion. However, using a gear hob with such a high number of starts, as the number of teeth of the generating gear, would be impractical. Instead, a gear hob with a reasonable number of starts (e.g., 1, 2, 3, or 4) should be used. In this case, the hob and hobbing machine setup parameters must be defined to achieve the desired enveloping helical gear tooth topography. Figure 10 illustrates the hobbing setup for an enveloping helical gear.

Finishing machining of enveloping helical gears (Figure 11) may involve tooth honing or grinding using a helical gear-shaped tool.

Other enveloping helical gear machining technologies, such as CNC milling (Figure 12), can also be considered.

6 SUMMARY

► The presented study of the load-carrying capacity of the enveloping crossed axis helical gears uses the Roark's formulas for the Hertz contact stress calculation.

► The study results indicate the possibility of considerable increase in load-carrying capacity achieved by replacing the convexo-convex tooth contact of the conventional crossed axis helical gears with the convexo-concave tooth contact of the enveloping crossed axis helical gears, with a significant

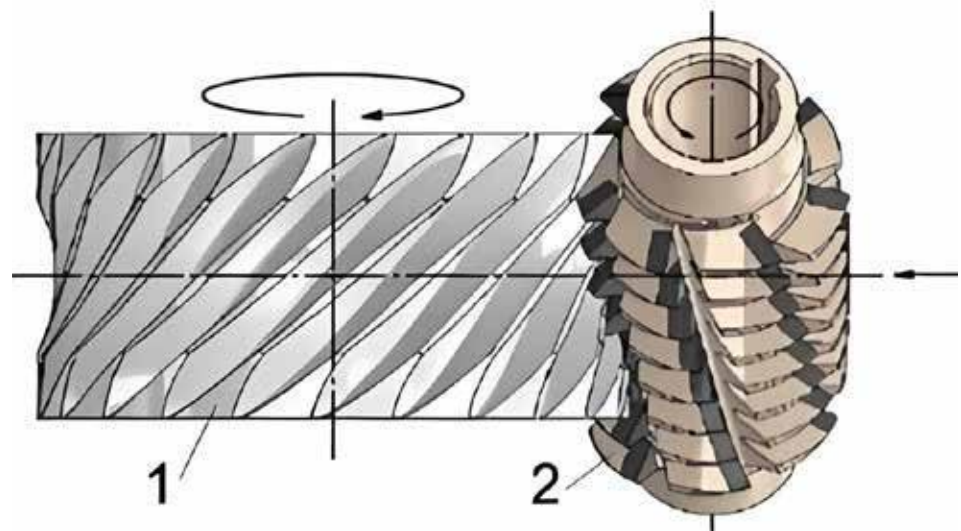


Figure 10: Enveloping helical gear hobbing; 1 – enveloping helical gear blank, 2 – gear hob.

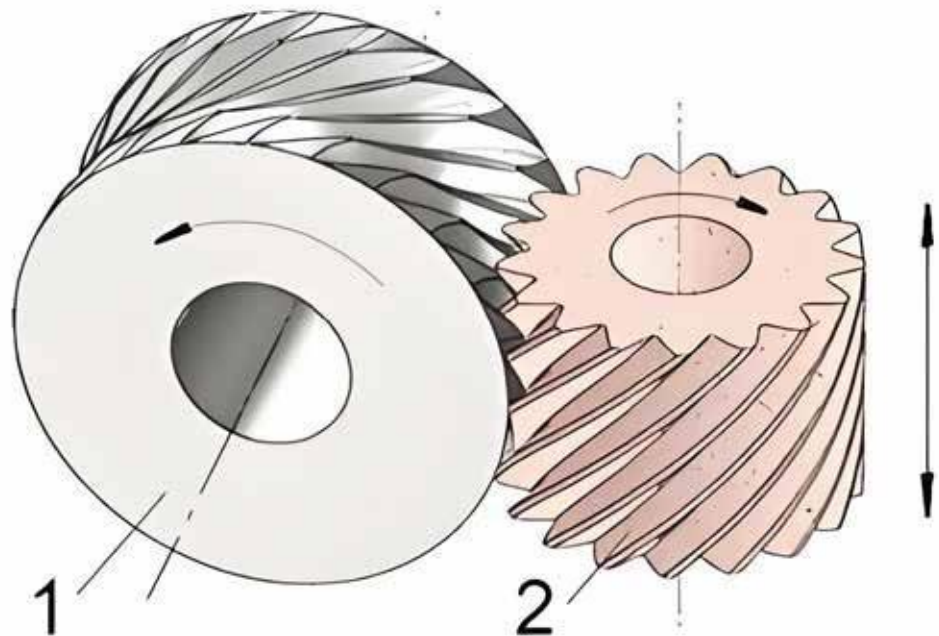


Figure 11: Enveloping helical gear honing; 1 – enveloping helical gear, 2 – gear honing or tooth grinding tool.

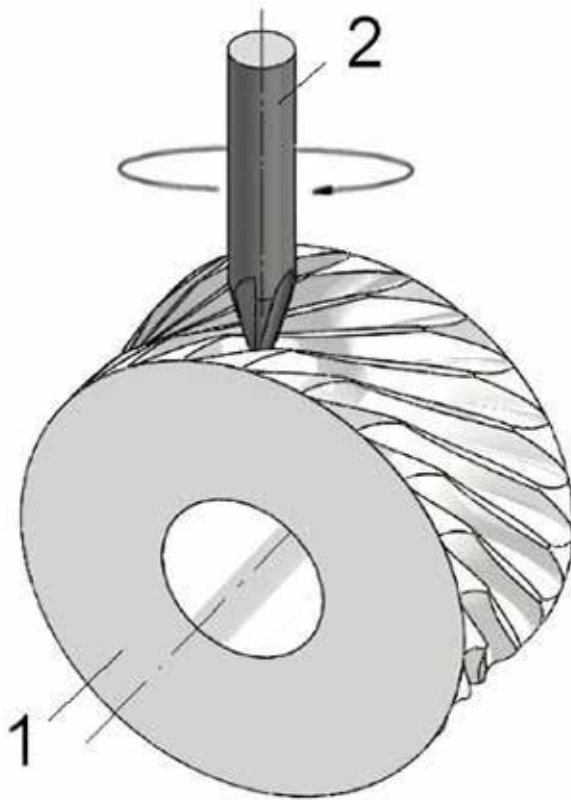


Figure 12: Enveloping helical gear CNC milling; 1 – enveloping helical gear, 2 – mill cutter.

increase in the tooth contact area.

► The BECAL and HyGEARS™ LTCA results of enveloping crossed axis helical gears validate the contact stress reduction and load-carrying capacity increase trends shown by the Roark's formulas.

► Enveloping crossed axis helical gears can be manufactured using

the same machining methods, machines, and tools as traditional helical and worm gears.

► Results of this study support the potential application of enveloping crossed axis helical gears in moderate to highly loaded gear drives. 📄

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ABOUT THE AUTHORS

Dr. Alexander Kapelevich is the founder of AKGears, LLC, a consulting firm specializing in gear design. He holds a Master's degree from the Moscow Aviation Institute and a Ph.D. from the Moscow State Technical University, both in Mechanical Engineering. Dr. Kapelevich is a developer of the modern Direct Gear Design® methodology and software. With over 40 years of experience in creating custom gear drives, his expertise includes gear transmission architecture, planetary systems, gear tooth profile optimization, asymmetric tooth gears, and maximizing gear drive performance. He is also the author of the books "Direct Gear Design" and "Asymmetric Gearing," as well as numerous technical articles.

Yefim Safris is an independent consultant and founder of Alphaenco, delivering advanced engineering and automation solutions. He holds an M.S. in Mechanical Engineering from St. Petersburg University. Safris began his career as a special machine tool engineer at Kurt Manufacturing and later worked as an automation engineer at Seagate Technology. He is an inventor on more than 20 U.S. patents.

Dr. Joachim Thomas currently works mainly as an independent developer and consultant with his company ZG Hypoid GmbH. After writing his doctoral thesis at FZG (Technical University of Munich), he worked for many years in industry as a gear expert specializing in bevel gears. He worked for MAN Truck & Bus, Klingelberg and Voith. He has been a member of the ISO standardization working group for bevel gears (ISO /TC60 /SC2 /WG13) since it was founded more than 25 years ago and has headed this group as convener since 2015. Joachim Thomas was co-author and co-editor of the textbook "Bevel Gear" by J. Klingelberg and has given lectures and training courses at numerous events worldwide.

Dr. Claude Gosselin has been involved in gear design and analysis for more than 40 years. He began as a gearbox designer at Pratt & Whitney Aircraft in Montreal, Canada; after completing his Ph.D., he taught Mechanical Engineering at Laval University, Quebec, Canada, as a full professor for some 20 years. As a consequence of his specialty in spiral bevel gears, Dr. Gosselin created the HyGEARS software, first introduced in Japan in 1994. HyGEARS offers to gear manufacturers an independent solution for gear design, analysis and manufacturing on conventional and CNC machines. He is now dedicated full time to the development and support of HyGEARS, which is used by 60-plus customers in some 12 countries.

Dr.-Ing. Wolf Wagner works at the Chair of Machine Elements of the Technical University of Dresden. Since 2014, he is part of the development team of the hypoid gear calculation software BECAL, which is developed for the FVA (Forschungsvereinigung Antriebstechnik e. V.). In 2024, he finished his doctoral thesis about the application of BECAL on hypoid gears in multibody system simulations.